School Mathematics Scope & Sequence (K through 12)
(Draft 2012)

(A document summarizing
the **Common Core State Standards** (CCSS)
with the list of key concepts, skills, and procedures
with interpretation and sequencing needed to be mastered
to implement the CCSS effectively and efficiently.)

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Math Scope & Sequence (Common Core & Sharma)

Common Core	K	1	2	Common Core K-12 Mathematical Practices
Big Ideas (Sharma)	**Know the ten numbers well and recognize (decomposition/re-composition) important and common objects.**	**Additive Reasoning:** Understand and learn additive reasoning; automatize 10 ×10 addition facts.	**Mastering Additive Reasoning (the inverse relationship between addition and subtraction—able to convert an addition into subtraction problem, fact, and statement**	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
By the end of the grade, students should be proficient with related math language and …		**Addition facts (10 ×10 grid)**	**Subtraction (10 × 10 grid) Tables (1, 2, 5, and 10)**	
Counting and Cardinality	• Know number names and the count sequence**. (at least to 100, forward & backward)** • Count to tell the number of objects. • Compare numbers.	**Count forward and back by 2 & 10 from any number**	**Count forward and back by 2, 5, 10 from any number** **Multiplication tables (1, 2, 5, 10) as counting**	
Operations and Algebraic Thinking K.OA	• Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	• Represent and solve problems involving addition and subtraction. • Understand and apply properties of operations and the relationship between addition and subtraction. • Add and subtract within 20. • Work with addition and subtraction equations.	• Represent and solve problems involving addition and subtraction procedures **(with & without borrowing)** • Add and subtract within 20. • Work with equal groups of objects to gain foundation for multiplication as repeated addition. **(Multiplication tables of 1, 2, 5, and 10),**	
Number and Operations in Base ten	• Work with numbers 11–19 to gain foundations for place value.	• Extend the counting sequence. • Understand place value**. (3 digit)** • Use place value understanding and properties of operations to add and subtract • **Read, write, express numbers in hundreds**	• Understand place value**. (read, write, express numbers in at least two cycles i.e. 100,000) – standard, semi-standard and expanded form** • **Apply place value to time & measurement** • Use place value understanding and properties of operations to add and subtract.	**Elements of effective mathematics teaching (Sharma):** 1. **Integrate language, concepts, and procedures of mathematical ideas** 2. **Know that mathematics is the study of patterns in quantity and space** 3. **Apply levels of knowing: intuitive, concrete, pictorial, abstract, application, and communication** 4. **Questioning Process: the quality of questions determines the effectiveness of teaching** 5. **Instructional models should range form discrete to continuous and should be exact, efficient, and elegant** 6. **Teacher must practice three roles: didactic, Socratic, and coaching**
Fractions		**Recognition of fractions (whole, halving, fourths)**	**Recognition of fractions (1/2, 1/3, ¼, 1/10)**	
Measurement and Data	• Describe and compare measurable attributes. • Classify objects and count the number of objects in categories. **(use body parts)**	• Measure lengths indirectly and by iterating length units**. (body parts & go between)** • Tell and write time. • Represent and interpret data.	• Measure and estimate lengths in standard units**. (using wholes & halves)** • Relate addition and subtraction to length. **(time, perimeter, and measurement)** • Work with time and money. • Represent and interpret data.	
Geometry	• Identify and describe shapes. • Analyze, compare, create, and compose shapes.	• Reason with shapes and their attributes. • **Recognize, describe and draw basic figures**	• Reason with shapes and their attributes. • **Recognize, describe and draw basic figures**	

Math Scope & Sequence Draft 2011 (Common Core & Professor M. Sharma)

Common Core	3	4	5	Common Core K-12 Mathematical Practices
Big Ideas (Sharma)	**Multiplicative Reasoning:** Transition from additive to multiplicative reasoning (following the mastery of the inverse relationships between addition and subtraction.)	**Mastering Multiplicative Reasoning (the inverse relationship between multiplication and division—able to convert an multiplication problem, fact, and statement into a division problem, fact or statement):**	**Proportional Reasoning:** Expand the idea of division to fractions; master the operations (addition, subtraction, multiplication, division) on fractions (fractions, decimals, percents) and their applications and understand the inverse relationship between the number of parts and the size of each part when making fractions from a whole.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
By the end of the grade, students should be proficient with related math language and …	**1. Tables 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12;** **2. Relate and automatize multiplication and division facts (fluency with understanding)** **3. Addition & Subtraction of Whole Numbers**	**1. Tables 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12;** **2. Relate and automatize multiplication and division facts with properties (commutative, associative, and distributive)**	**1. Multiplication and Division of Whole Numbers** **2. Identify and represent fractions and decimals, and compare them on a number line or with other common representations** **3. Fractions and decimals and common percents, and with the addition and subtraction of fractions and decimals.** **4. Solve problems involving perimeter and area of triangles and all quadrilaterals having at least one pair of parallel sides**	**Elements of effective mathematics teaching (Sharma):** **1. Integrate language, concepts, and procedures** **2. Know that mathematics is the study of patterns in quantity and space** **3. Levels of knowing: intuitive, concrete, pictorial, abstract, application, and communications** **4. Questioning Process: the quality of questions determines the effectiveness of teaching** **5. Instructional models should range form discrete to continuous and should be exact, efficient, and elegant** **6. Teacher must practice three roles: didactic, Socratic, and coaching** **7. With proper scaffolding a teacher can fill the gaps in a student's skills and concepts and take the child to a higher level of thinking.** **8. Effective scaffolding involves: common goals (teacher and student), continuous formative assessment, proper questioning, and fading**
Counting and Cardinality	**• Count forward and back by 2, 5, 10, 100, 1/2 from any number**	**Counting forward and backward by 2, 5, 10, 1/2, 1/3, 1/4, 1/5, 1/10 from any given number**	**Counting forward and backward by 2, 5, 10, 1/2, 1/3, 1/4, 1/5, 1/10 or .1 from any given number**	
Operations and Algebraic Thinking	• Represent and solve problems involving multiplication and division. • Understand properties of multiplication and the relationship b/w multiplication and division. • Multiply and divide within 100. • Solve problems involving the four operations, and identify and explain patterns in arithmetic	• Master and use the four operations with whole numbers to solve problems. • Gain familiarity with factors and multiples. • Generate and analyze numerical and spatial patterns. • **Operations on multi-digit numbers (addition, subtraction, multiplication, and division) with and without borrowing**	•Write and interpret numerical expressions. • Analyze patterns and relationships. • **Divisibility rules for 2, 3, 4, 5, 6, 8, 9, and 10**	
Number and operations in base ten	Use place value understanding and properties of operations to perform multi-digit arithmetic. **(Red, write, express numbers in any # of cycles in standard, semi-standard and expanded forms)**	• Generalize place value understanding for multi-digit whole numbers. **Express in standard, semi-standard and expanded forms** • Use place value understanding and properties of operations to perform multi-digit arithmetic. **(Read, write, express numbers in any # of cycles and simple decimals**	• Understand the place value system.for **any digit numbers (Express in standard, semi-standard and expanded form)** • Perform operations with multi-digit whole numbers and with decimals to hundredths. **(Addition, subtraction, multiplication, and division with and without borrowing)** **(Read, write, express numbers in any # of cycles and decimal #s)**	
Number and operations – fractions	• Develop understanding of fractions as numbers by locating factions on the number line.	• Extend understanding of fraction equivalence and ordering**.** • Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. **(add/subtract simple fractions)** • Understand decimal notation for fractions, and compare decimal fractions.	• Use equivalent fractions as a strategy to add and subtract fractions. • Apply and extend previous understandings of multiplication and division to multiply and divide fractions. • **Relate fractions, decimals, percents, and ratio; Operations on**	
Measurement and data	• Solve problems involving measurement and estimation of intervals (time, liquid volume, mass) • Represent and interpret data. • Geometric measurement: understand concepts of area and relate to multiplication and addition and recognize perimeter as an attribute of plane figures, distinguish between linear and area measures.	• Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. **(standard units whole & halves)** • Represent and interpret data. • Geometric measurement: understand concepts of angle and measure angles.	• **Measurement using standard units (correct up to tenths)** • Convert like measurement units within a given measurement system. • Represent and interpret data **(make inferences)** • Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. **Measure any polygon**	
Geometry	• Reason with shapes and their attributes. • **Recognize, describe and draw basic figures & their relationships**	Draw and identify lines and angles, and classify shapes by properties of their lines and angles.	• Use coordinate plane to solve real-world and math problems. • Classify plane figures (classify quadrilaterals) into categories from properties. • **Recognize, describe, and draw basic figures and their relationships; calculate perimeter of polygons and area of rectangles, triangles, and derived shapes**	

Math Scope & Sequence Draft 2011 (Common Core & Professor M. Sharma)

Common Core	6	7	8	Common Core K-12 Mathematical Practices
Big Ideas (Sharma)	**Proportional Reasoning:** Consolidate the mastery of operations on fractions and expand to ratio, proportion, rate, scale factor; Expand the Number system to include integers, rational numbers and apply proportional reasoning	**Big Idea—Algebraic Reasoning:** concept of variability; Consolidate proportional reasoning; extending arithmetic reasoning to generalizations—rate of change, slope, scale factor, transformations; fluency in operations on rational numbers	**Algebraic Thinking** and **Modeling:** Consolidate algebraic thinking; Operations on algebraic expressions; Mastering linear equations; understanding non-linear relationships such as quadratic expressions and forming and working with equations; Transformations between representations—tabular, graphing, patterns, equations.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
By the end of the grade, students should be proficient with related math language and …	**1. Multiplication and division of fractions and decimals.** **2. All operations involving positive and negative integers** 3. **Analyze the properties of two dimensional shapes and solve problems involving perimeter and area,** **4. Analyze the properties of three-dimensional shapes and solve problems involving surface area and volume.**	**1. All operations involving rational numbers.** **2. Solving problems involving percent, ratio, and rate and extend this work to proportionality.** **3. Relationships between similar triangles and the concept of the slope of a line.**		
Counting and Cardinality	**Counting forward and backward by any number (whole, fraction, decimal) from any given number)**	Locating numbers (whole numbers, integers, rationals) on number line	Locating numbers (whole, integers, rationals, and real numbers)	
Ratios and Proportional Relationships	• Understand ratio concepts and use ratio reasoning to solve problems. • **Relate fractions, decimals, percents, and ratio; operations on**	Analyze proportional relationships and use them to solve real-world, mathematical problems, and perform dimensional analysis	Generalizing the idea of ratio and proportion to linear and distinguish it from non-linear relationships	
The Number System	• **Reading, writing, and expressing numbers in any number of cycles and decimal numbers** • **Place value any digit number including decimals (standard, semi-expanded, expanded and exponential) forms** • Apply and extend previous understandings of multiplication and division to division of fractions • Compute fluently with multi-digit numbers and find common factors and multiples. • Apply and extend previous understandings of numbers to the system of rational numbers.	• Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. • **Place value any digit number including decimals (standard, semi-expanded, expanded, and exponential) forms**	• Know that there are numbers that are not rational and approximate irrationals by rational numbers (terminating, non-terminating, and repeating decimals)	**Elements of effective mathematics teaching (Sharma):** 1. **Integrate language, concepts, and procedures** 2. **Know that mathematics is the study of patterns in quantity and space** 3. **Levels of knowing: intuitive, concrete, pictorial, abstract, application, and communications** 4. **Questioning Process: the quality of questions determines the effectiveness of teaching** 5. **Instructional models should range form discrete to continuous and should be exact, efficient, and elegant** 6. **Teacher must practice three roles: didactic, Socratic, and coaching** 7. **With proper scaffolding a teacher can fill the gaps in a student's skills and concepts and take the child to a higher level of thinking.** 8. **Effective scaffolding involves: common goals (teacher and student), continuous formative assessment, proper questioning, and fading**
Expressions & Equations	• Apply and extend previous understandings of arithmetic to algebraic expressions and operations • Reason about and solve one-variable equations and inequalities. • Represent and analyze quantitative relationships between dependent & independent variables.	• Use properties of operations to generate equivalent expressions • Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	• Work with radicals and integer exponents. • Understand the connections between proportional relationships, lines, and linear equations. • Analyze and solve linear equations and simultaneous linear equations using various methods.	
Functions		Expand the ideas of fraction machines to relations and functions	• Define, evaluate, and compare functions. • Use functions to model relationships b/w quantities.	
Geometry	• Solve real-world and mathematical problems involving area, surface area, and volume. • **Recognizing, describing, and drawing all the basic figures and their relationships; calculate perimeter of any polygon and circle and area of rectangles, triangles, and derived shapes made of these shapes and circles; genealogy of quadrilaterals and triangles**	• Draw, construct and describe geometrical figures and describe the relationships between them. • Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	• Understand congruence and similarity using physical models, transparencies, or geometry software. • Understand and apply the Pythagorean theorem. • Solve real-world and mathematical problems involving volume of cylinders, cones &spheres.	
Probability & Statistics	• Develop understanding of statistical variability. • Summarize and describe distributions. • **Representations of data and making inferences**	• Use random sampling to draw inferences about a population. • Draw informal comparative inferences about two populations. • Investigate chance processes and develop, use, and evaluate probability models.	• Investigate patterns of association in bivariate data.	

Math Scope & Sequence Draft 2011 9th through 11th Grade (Common Core & Sharma)

Quantitative thinking and Its Representations	Arithmetic of Algebra and its Representations	Functions and Operations on Functions	Spatial Thinking and Geometrical Representations	Discrete Models: Statistics and Probability
The Real Number System • Extend the properties of exponents to rational exponents • Master arithmetic operations on and use properties of rational and irrational numbers in solving problems **Quantities** • Reason quantitatively and use units and dimensional analysis to solve problems **The Complex Number System** • Perform arithmetic operations on the set of complex numbers • Represent complex numbers and their operations on the complex plane • Use complex numbers in polynomial identities and equations **Vector and Matrix Quantities** • Represent and model with vector quantities. • Perform operations on vectors. • Perform operations on matrices and use matrices in applications.	**Structures in Algebraic Expressions** • Interpret the structure of algebraic expressions • Write expressions in equivalent forms to solve problems **Arithmetic of Polynomials and Rational Expressions** • Perform arithmetic operations on polynomials • Understand the relationship between zeros and factors of polynomials; study the nature of functions and their graphs (zeroes, asymptotes, singularities, etc.) • Use polynomial identities to solve problems • Rewrite rational expressions **Creating Equations and Modeling Problems** • Create equations that describe numbers, relationships, and problem situations **Reasoning with Equations and Inequalities** • Understand solving equations as a process of reasoning and explain the reasoning • Solve equations and inequalities in one variable • Solve systems of equations using different methods • Represent and solve equations and inequalities graphically and algebraically	**Interpreting Functions** • Understand the concept of a function and use function notation • Interpret functions that arise in applications in terms of the context • Analyze functions using different representations and with respect to their domain and range **Building Functions** • Build a function that models a relationship between two quantities • Build new functions from existing functions (By adding, subtracting, multiplying, dividing, and composing functions) **Linear, Quadratic, and Exponential Models** • Construct and compare linear, quadratic, and exponential models and solve problems • Interpret expressions for functions in terms of the situation they model **Trigonometric Functions** • Extend the domain of trigonometric functions using the unit circle • Model periodic phenomena with trigonometric functions • Prove and apply trigonometric identities **Special Functions: Piecewise, step, integer, absolute function**	**Congruence** • Experiment with transformations in the plane • Understand congruence in terms of rigid motions • Prove geometric theorems • Make geometric constructions **Similarity, Right Triangles, and Trigonometry** • Understand similarity in terms of similarity transformations • Prove theorems involving similarity • Define trigonometric ratios and solve problems involving right triangles • Apply trigonometry to general triangles **Circles** • Understand and apply theorems about circles • Find arc lengths and areas of sectors of circles **Expressing Geometric Properties with Equations** • Translate between the geometric description and the equation for a conic sections (parabola, ellipse, circle, hyperbola, pair of straight lines) • Use coordinate systems to prove simple geometric theorems algebraically **Geometric Measurement and Dimension** • Explain volume formulas and use them to solve problems • Visualize relationships between two dimensional and three-dimensional objects **Modeling with Geometry** • apply geometric concepts in modeling situations	**Interpreting Categorical and Quantitative Data** • Summarize, represent, and interpret data on a single count or measurement variable • Summarize, represent, and interpret data on two categorical and quantitative variables • Interpret linear models **Making Inferences and Justifying Conclusions** • Understand and evaluate random processes underlying statistical experiments • Make inferences and justify conclusions from sample surveys, experiments and observational studies **Conditional Probability and the Rules of Probability** • Understand independence and conditional probability and use them to interpret data • Use the rules of probability to compute probabilities of compound events in a uniform probability model **Using Probability to Make Decisions** • Calculate expected values and use them to solve problems • Use probability to evaluate outcomes of decisions

Modeling:
Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. Modeling links classroom mathematics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data. A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Common Core K-12 Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Elements of Effective Mathematics Teaching (Sharma)

1. **Integrate language, concepts, and procedures of mathematics**
2. **Know that mathematics is the study of patterns in quantity and space**
3. **Levels of knowing a mathematics idea: intuitive, concrete, pictorial, abstract, application, and communications**
4. **Questioning Process: the quality of questions determines the effectiveness of teaching**
5. **Instructional models should range form discrete to continuous and should be exact, efficient, and elegant**
6. **Teacher must practice three roles: didactic, Socratic, and coaching**
7. **With proper scaffolding a teacher can fill the gaps in a student's skills and concepts and take the child to a higher level of thinking.**
8. **Effective scaffolding involves: common goals (teacher and student), continuous formative assessment, proper questioning, and fading**